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Design with Multiple-Delay-Model and Multiple-Design-Point Approach

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In this paper, the multiple-delay-model and multiple-design-point (MDM/MDP) approach is applied to the AIAA 1992 Controls Design Challenge. Design goals of the flight control system are presented with standard quadratic performance indices, and linear control laws of constant feedback gains are obtained numerically with parameter optimization. Robustness against uncertainties is guaranteed by giving appropriate MDMs, and robustness against changing dynamics is obtained by selecting MDPs. Trade-offs between control performance and robustness are conducted with MDMs. Simple PIs introduce reasonable feedback control laws. Realistic control laws are derived from the analytical results. Time responses of the two given flight missions and performances at four evaluation points demonstrate feasibility of the simple constant-gain feedback control.

Introduction

ROBUST flight control systems design has been discussed from various approaches. The multiple-model (MM) approach applied in this paper is one of the robust linear control system design methods. The MM approach is an extension of the standard linear quadratic regulator (LQR), in which the control system design goal is presented with a performance index (PI) of quadratic functions, and the optimal control law is obtained by minimizing the PI. Although the quadratic PI is a good measure of the flight control performance, the LQR does not necessarily give favorable practical control laws when it neglects the difference between a given mathematical model and an actual dynamics. To cope with this problem, the LQR is extended using a concept of multiple models. Uncertainties and/or changing parameters are represented by multiple models, and the total PI is minimized.[1–8]

The MDM approach proposed by the author uses delay models to represent high-frequency uncertainties.[9,10] Response due to control is causal and has some inevitable delay, which is often neglected in the standard LQR but is quite important when high-control performance is required. Stability and performance against delay is an important measure of the control system robustness, and the optimal control introduced with the measure, or from MDMs, gives a reasonable trade-off between the control performance and robustness. In the MDM approach, two first-order delay models are used for each control input, one having no delay and the other having maximum possible delay. The MDM approach with output proportional feedback of structured gains introduces a simple control law that is easily implemented. The approach naturally gives moderate control frequency bandwidths and saves tedious adjustment of weighting parameters on the control cost. The MDP method that has been frequently used in flight control problems is also applied in order to investigate the feasibility of a constant-gain feedback controller.

Multiple-Delay-Model and Multiple-Design-Point Approach

Mathematical models and PIs are very important in analytical control system design. These are discussed in this section, showing how the MDM/MDP approach is applied in the Controls Design Challenge.[11]

Linear Time-Invariant Model at Each Flight Condition

Standard linear time-invariant differential equations of decoupled longitudinal and lateral-directional motions were used for the control system design and analysis. The linearized equations were derived for each design point of Mach number and altitude. Eighteen points were selected for the control system design, and four evaluation points have already been defined in the program. Totally 22 points are considered. Table 1 shows the trimmed values at each point, where points A–D correspond to the four evaluation points. Figure 1 shows these points in the flight envelope. Equilibrium condition was realized in the given numerical simulation program by a simple autopilot, and stability and control derivatives were numerically calculated with a small perturbation. The following linear differential equations of longitudinal and lateral-directional motions were used:

$$\begin{aligned}\frac{\mathrm{d}x_{\mathrm{lon}}}{\mathrm{d}t} &= A_{\mathrm{lon}}x_{\mathrm{lon}}(t) + B_{\mathrm{lon}}u_{\mathrm{lon}}(t)\\ y_{\mathrm{lon}}(t) &= C_{\mathrm{lon}}x_{\mathrm{lon}}(t)\end{aligned} \tag{1}$$

$$\begin{aligned}\frac{\mathrm{d}x_{\mathrm{lat}}}{\mathrm{d}t} &= A_{\mathrm{lat}}x_{\mathrm{lat}}(t) + B_{\mathrm{lat}}u_{\mathrm{lat}}(t)\\ y_{\mathrm{lat}}(t) &= C_{\mathrm{lat}}x_{\mathrm{lat}}(t) + D_{\mathrm{lat}}u_{\mathrm{lat}}(t)\end{aligned} \tag{2}$$

where $x_{\mathrm{lon}} = [u, \alpha, q, \theta, h]$, $u_{\mathrm{lon}} = [\delta_e, \delta_t]$, $y_{\mathrm{lon}} = [q, \theta, \mathrm{d}h/\mathrm{d}t, h, M]$, $x_{\mathrm{lat}} = [\beta, p, r, \phi]$, $u_{\mathrm{lat}} = [\delta_a, \delta_r]$, and $y_{\mathrm{lat}} = [p, \phi, r, a_y]$. Here, u is velocity, α attack angle, q pitch rate, θ pitch attitude, δ_e symmetric stabilator angle, δ_t thrust, h inertial altitude, $\mathrm{d}h/\mathrm{d}t$ altitude rate, M Mach number, β side slip angle, p roll rate, r yaw rate, ϕ bank angle, δ_a aileron angle, δ_r rudder angle, and a_y lateral

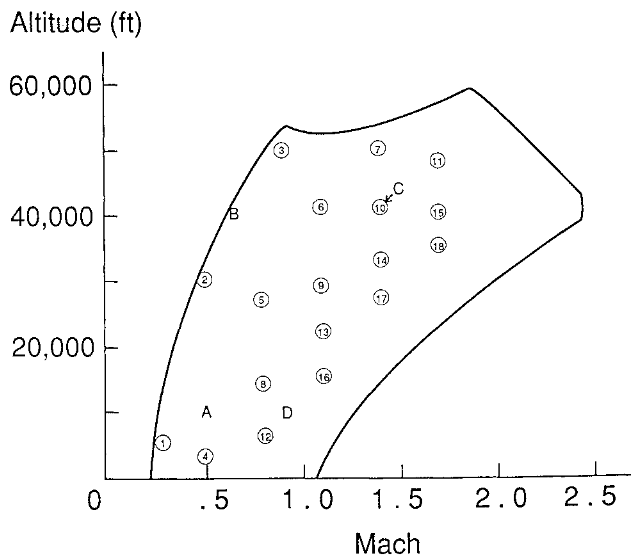

Fig. 1 Flight envelope and design points.

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Table 1 Design points and evaluation points

Point number	Mach number	Altitude, ft	Dynamic pressure, lb/ft	Alpha, deg	Stabilator, deg	Thrust, lb
1	0.3	5,000	113	11.0	−7.51	7,450
2	0.5	30,000	117	10.8	−7.08	7,440
3	0.9	50,000	169	7.16	−4.56	5,510
4	0.5	3,000	353	3.52	−2.20	4,870
5	0.8	27,000	379	3.30	−1.26	4,420
6	1.1	40,000	446	3.54	−7.47	9,470
7	1.4	50,000	500	4.10	−5.69	12,200
8	0.8	14,000	653	1.92	−0.585	6,590
9	1.1	29,000	746	2.20	−3.78	15,650
10	1.4	40,000	807	2.46	−1.58	17,420
11	1.7	48,000	866	2.66	−1.58	15,570
12	0.8	6,000	890	1.41	−0.314	10,620
13	1.1	22,000	1014	1.66	−2.31	18,600
14	1.4	33,000	1124	1.70	0.340	23,200
15	1.7	40,000	1269	1.73	0.568	23,400
16	1.1	15,000	1358	1.29	−1.24	21,800
17	1.4	27,000	1478	1.24	1.52	27,700
18	1.7	35,000	1613	1.30	1.54	30,000
A	0.5	9,800	273	4.58	−2.87	4,580
B	0.6	39,800	110	11.6	−7.20	8,670
C	1.4	39,800	815	2.43	−1.52	16,900
D	0.9	9,800	1014	1.25	−0.57	10,400

acceleration. All variables except p, q, r, and $\mathrm{d}h/\mathrm{d}t$ are deviations from the trimmed values. A linearly linked differential stabilator with aileron is simply assumed, i.e., $\delta_d = \frac{1}{2}\delta_a$, and its effect is included in the aileron control derivative. Linearization is not essential for the MDM/MDP approaches, but it significantly saves computational time in evaluating the closed-loop system stability and the PI values.

Performance Indices

The control system design and analysis were performed with quadratic PIs, which are commonly used in the standard LQR.[12,13] If the defined quadratic PI is an appropriate measure for the control performance, the optimal control that minimizes the PI naturally achieves the design goal. Considering the Controls Design Challenge program tasks, it is assumed that the design goal is presented with the following PIs for the two rigid-body motions:

Longitudinal motion:

$$J_{\mathrm{lon}_i} = E\left[\int_0^\infty \left\{\left(\frac{h_i(t) - h_c}{h_0}\right)^2 + \left(\frac{M_i(t) - M_c}{M_0}\right)^2\right\}\mathrm{d}t\right] \tag{3}$$

It is defined from the given flight tasks that altitude and Mach number are the variables to be controlled with stabilator and power. The performance index (3) shows this design goal; i.e., quadratic function time integrals of these errors from the commands are evaluated, where subscripts i, c, and 0 denote variables of the ith model, constant command, and appropriately defined normalization constants, respectively.[12] Here, h_0 for deviations of altitude and M_0 for Mach number are design parameters, and for the control design the same energies are chosen, i.e., $M_0 = gh_0/aU_0$, where a is the speed of sound and U_0 is the true airspeed. The average is denoted by $E[\,]$. Stochastic presentation of the PI is due to randomness of the assumed initial condition. It is simply defined that all states are initially zero and only constant commands are randomly distributed, variances of which are given as $E[(M_c/M_0)^2] = 1$ and $E[(h_c/h_0)^2] = 1$, and M_c and h_c are uncorrelated. This initial condition corresponds to no external disturbances such as gust and measurement noise, except for command inputs.

Lateral-directional motion:

$$J_{\mathrm{lat}_i} = E\left[\int_0^\infty \left\{\left(\frac{\phi_i(t) - \phi_c}{\phi_0}\right)^2 + \left(\frac{a_{yi}(t) - a_{yc}}{a_{y0}}\right)^2\right\}\mathrm{d}t\right] \tag{4}$$

For lateral-directional motion, roll attitude and lateral acceleration are to be controlled with aileron, differential stabilator, and rudder. Normalization constants ϕ_0 and a_{y0} are design parameters, which were simply given as $\phi_0 = a_{y0}$. As it is defined for the longitudinal motion, the initial condition is assumed that all states are zero and only command inputs are randomly distributed, variances of which were given as $E[(\phi_c/\phi_0)^2] = 1$ and $E[(a_{yc}/a_{y0})^2] = 1$, and they are uncorrelated. When coordinated flight is required, the lateral acceleration command a_{yc} is always zero, but a nonzero command is given in order to suppress a bias error for the lateral acceleration.

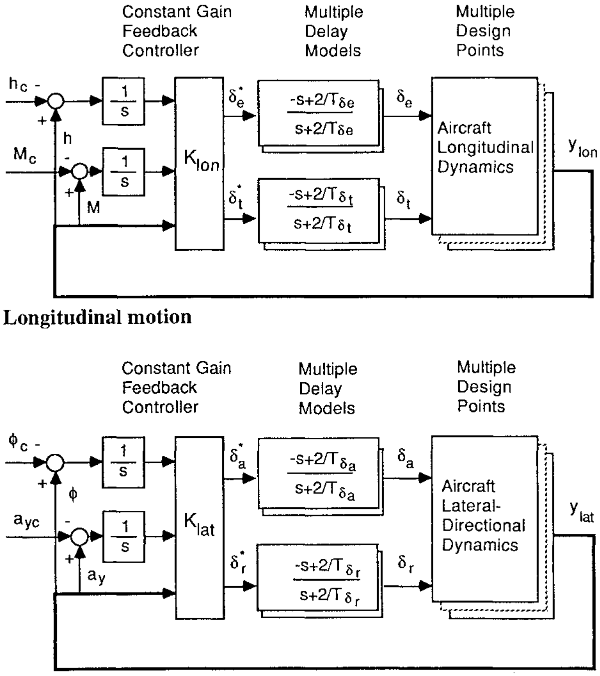
In the standard LQR case, control costs should be considered, but in the MDM approach the control performance can be directly defined without control costs. The performance indices (3) and (4) have a dimension of time, or second. Since the quadratic function in the time integrals has a value of 2 at the initial time ($t = 0$), the value of the PI is a time constant of the control system, or quite roughly, it indicates a settling time to command inputs.

Multiple-Delay Models

As the MDM approach has been discussed in Refs. 9 and 10; an outline is briefly presented here. The MDM approach is an application of the MM. Different delay models are used to present high-frequency uncertainty. The MDM approach was proposed to introduce natural modeling of inevitable high-frequency phase uncertainty. The measured response due to control input has some delay, causes of which are widely spread, such as actuator and sensor dynamics, digital signal transferring and processing, and noise filter. High-frequency dynamics are difficult to be modeled, and therefore the high-frequency uncertainty gives a limit on the control performance, or the feedback control frequency bandwidth.[14] The high-frequency phase uncertainty can introduce a simple trade-off between control performance and robustness. The MDM approach uses the following first-order uncertain delay model that is inserted in the control input:

$$\delta_i(s) = \frac{-T_{\delta_i}s/2 + 1}{T_{\delta_i}s/2 + 1}\delta_i^*(s), \qquad i = 1, 2 \tag{5}$$

Two delay models are used, i.e., $T_{\delta_1} = 0$, $T_{\delta_2} = T_{\delta_0}$; one has no delay and the other has T_{δ_0} delay, where T_{δ_0} is an appropriately given design parameter that can be introduced from the maximum possible delay time in the control loop. The gain of frequency response (5) is constantly 0 dB and the phase delay increases along with frequency from 0 to 180 deg and becomes 90 deg at frequency $2/T_{\delta_0}$. Figures 2a and 2b show block diagrams for the control system analysis. Multiple delay models (5) represent uncertainties of each control input response.



a) Longitudinal motion

b) Lateral-directional motion

Fig. 2 System design block diagram.

Structured Control Law and Total Performance Index

With the MDM approach, the control structure should be predetermined in order to avoid mathematical difficulty in the optimal solution as well as to introduce practically feasible results. The standard LQR result gives a good reference for the control structure: i.e., full state including time integrals of command errors are to be fed back. Considering directly accessible variables, a few state variables are replaced by measured variables, and the following full-output feedback control structure is assumed:

$$u_{\text{lon}}^*(t) = K_{\text{lon}}\xi_{\text{lon}}(t)$$
$$u_{\text{lon}}^* = [\delta_e^*, \delta_t^*]^T \tag{6}$$
$$\xi_{\text{lon}} = \left[q, \theta, \frac{\mathrm{d}h}{\mathrm{d}t}, h, \int [h(t) - h_c]\mathrm{d}t, M, \int [M(t) - M_c]\,\mathrm{d}t\right]^T$$

$$u_{\text{lat}}^* = K_{\text{lat}}\xi_{\text{lat}}(t)$$
$$u_{\text{lat}}^* = [\delta_a^*, \delta_r^*]^T \tag{7}$$
$$\xi_{\text{lat}} = \left[p, \phi, \int [\phi(t) - \phi_c]\,\mathrm{d}t, r, a_y, \int [a_y(t) - a_{yc}]\,\mathrm{d}t\right]^T$$

A Mach number corresponding to velocity is used. Vertical speed $\mathrm{d}h/\mathrm{d}t$, which is not a directly accessible variable, is easily reconstructed from inertial altitude and motion sensor outputs. Directly accessible lateral acceleration is used for side slip angle β, and coordinated turn condition is given by $a_y = 0$.

The feedback gains K_{lon} and K_{lat} are designed by minimizing the following total PIs:

$$J_{\text{lon}} = \frac{1}{N}\sum_{i=1}^{N} J_{\text{lon}_i} \tag{8}$$

$$J_{\text{lat}} = \frac{1}{N}\sum_{i=1}^{N} J_{\text{lat}_i} \tag{9}$$

The total PI is a simple average of all PIs of each different delay model and each different design point, where N is the number of all considered models.

Control system analysis was carried out with this full-output feedback controller (6) and (7) for each design point. In many cases, it is not necessary to close all loops, and partial loop closure, or structured feedback gain, can give reasonable control performance. Such structured gain feedback controls are also investigated.

Multiple Design Points

As shown in Fig. 1 and Table 1, 18 design points were selected for the control law design. Control laws that minimize the MDM performance indices at each single design point are calculated, and the obtained PIs are compared. Then, control laws that minimize the total PI of all $72(2 \times 2 \times 18)$ models, two delay models for two inputs at 18 design points, are calculated. Feasibility of the constant-gain controller is studied.

Results

Control system analysis and design were conducted for longitudinal and lateral-directional motions separately. First, control laws that minimize the MDM control PI at each single design point are calculated for three types of feedback gain structure. Four delay models, two delay models for two control inputs, are considered, i.e., $[T_e, T_t] = [0, 0], [0, 2], [0.4, 0], [0.4, 2]$ for longitudinal motion, $[T_a, T_r] = [0, 0], [0, 0.4], [0.4, 0], [0.4, 0.4]$ for lateral-directional motion. Figure 3 shows the minimized MDM performance index, an average PI of four delay models, for longitudinal motion. Control law 1 is full-output feedback control; i.e., all elements of the K_{lon} matrix are optimized for each design point. Control law 2 opens the loops from pitch and altitude to thrust; i.e., all outputs are fed back to the stabilator, but only the Mach number is fed back to thrust. Control law 3 has no cross feedback; i.e., pitch and altitude are fed back to the stabilator, and the Mach number is fed back to thrust. Comparing the obtained performances, each design point gives similar control performance. This indicates that the MDM control laws derived from the same delay models have similar time constants. Performances with control law 2 are not significantly different from those with control law 1, and they are slightly better than with Control law 3. Next, a control law that minimizes the total MDM performance index of all 18 design points is calculated. Control law 3 type gain structure is considered. Figure 4 shows the MDM/MDP control performances at each design point. The plotted MDM performance index at each point is normalized by that obtained with each optimal control law 3. It is shown that the constant-gain controller stabilizes all delay models at all design points except point D, which is not explicitly considered in the design, and normalized values of the control performances are less than two; that is, the performance is not so significantly different from that obtained with the optimal control law at each point. Furthermore, there is no definite tendency of flight condition dependency in the normalized performance indices.

Figures 5 and 6 are performance results for lateral-directional motion. Figure 5 shows the optimal MDM performances obtained at each single point. Control law 1 is full output feedback as well. Control law 2 opens the loop from roll attitude to rudder. Control law 3 has no cross feedback; i.e., roll attitude is fed back to aileron and

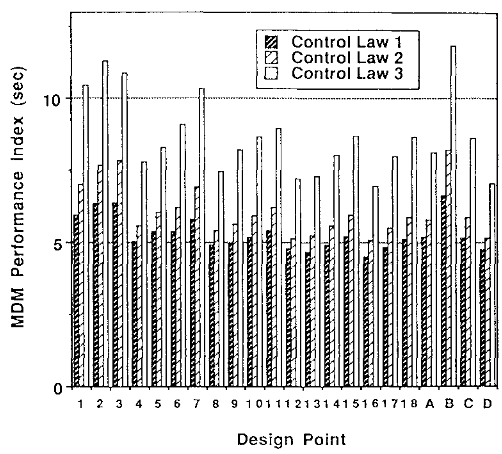


Fig. 3 MDM performance index designed at each point: longitudinal motion.

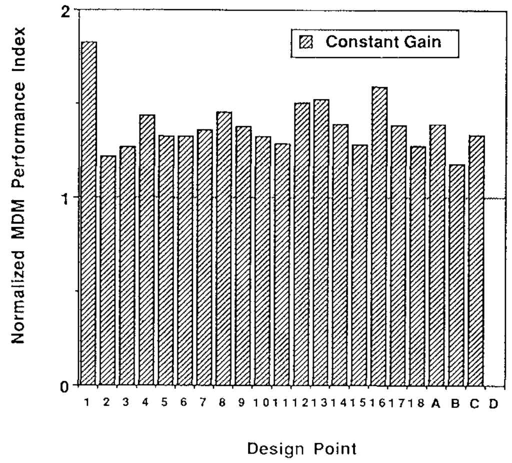


Fig. 4 MDM performance index of MDM/MDP control law: longitudinal motion.

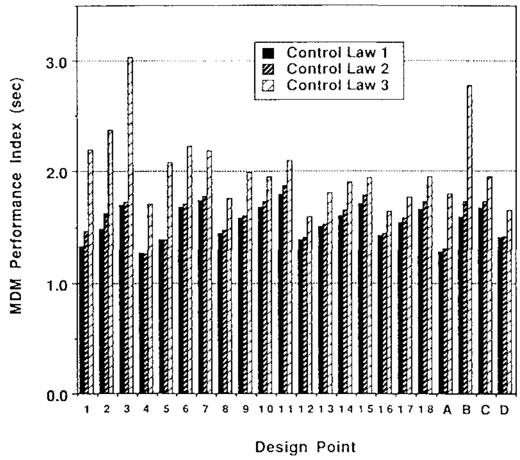


Fig. 5 MDM performance index designed at each point: lateral-directional motion.

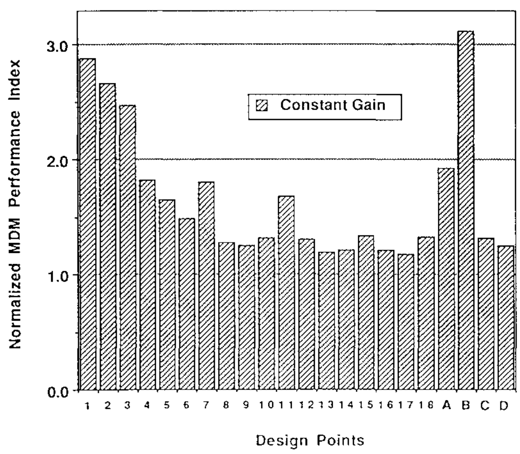


Fig. 6 MDM performance index of MDM/MDP control law: lateral-directional motion.

differential stabilator, and yaw rate and lateral acceleration are fed back to rudder. As the situation is similar for the longitudinal motion, there is no significant difference for each design point, and control law 2 gives performances close to the more complex control law 1. The control law 2 type gain structure is adopted for a constant gain controller. Figure 6 shows results of performances of each point with the MDM/MDP control law designed from 72 models. The plotted MDM performance index is normalized with the minimized value at each point. It is shown that the constant-gain control law stabilizes all delay models at all design points, and the normalized PIs vary from 1.3 to 3 depending on the flight condition. At low

Table 2 Feedback gains of MDM/MDP control law

K_{lon} =						
0.0614	0.121	0.00108	0.398×10^{-3}	0.38×10^{-4}	0	0
0	0	0	0	0	−19.5	−3.75
K_{lat} =						
−0.139	−0.386	−0.311	−0.127	−0.0445	0.275	
0	0	0	0.477	−0.0298	0.243	

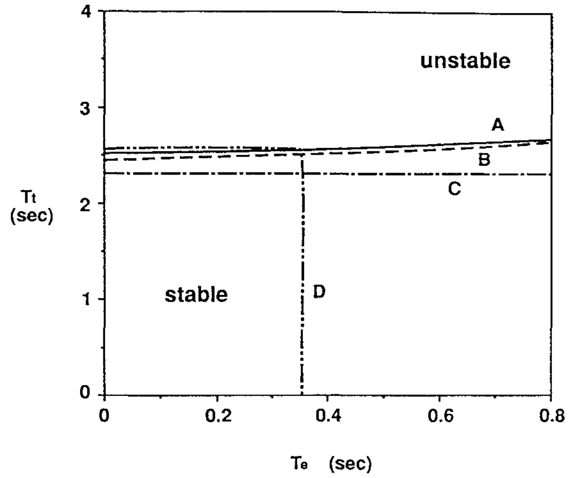


Fig. 7 Stability boundaries at four evaluation points: longitudinal motion.

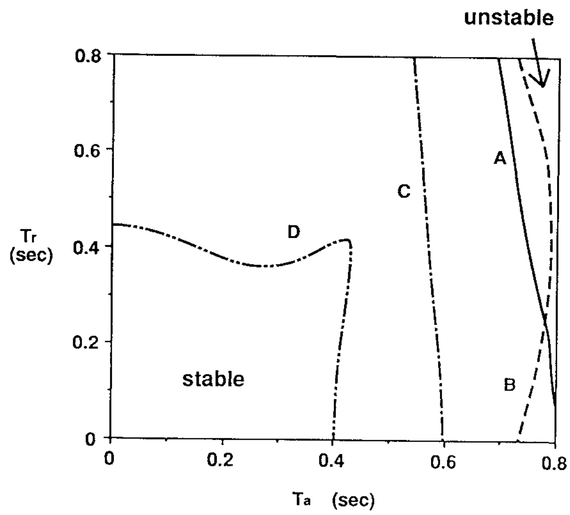


Fig. 8 Stability boundaries at four evaluation points: lateral-directional motion.

dynamic pressure points, it is worse than those at high dynamic pressure points.

For both longitudinal and lateral-directional motions, the obtained MDM/MDP constant-gain control laws are adopted as the final control laws, feedback gains of which are listed in Table 2.

Figures 7 and 8 show stability boundaries against delay times in the control loop at the four evaluation points A–D for longitudinal and lateral-directional motions, respectively. The final MDM/MDP control laws in Table 2 are used. Although the control laws are derived from 18 design points, in which the four evaluation points are not included, control systems' stability is guaranteed against a wide range of delay times.

Performance index values at four evaluation points are listed in Table 3. PI (one-half delay) is the performance obtained by assuming half delay times for each control input, i.e., $[T_e, T_t] = [0.2, 1]$ and $[T_a, T_r] = [0.2, 0.2]$. The MDM performances that are averaged performance indices of four delay models are similar to those of half delay times. The PI roughly indicates a time constant of

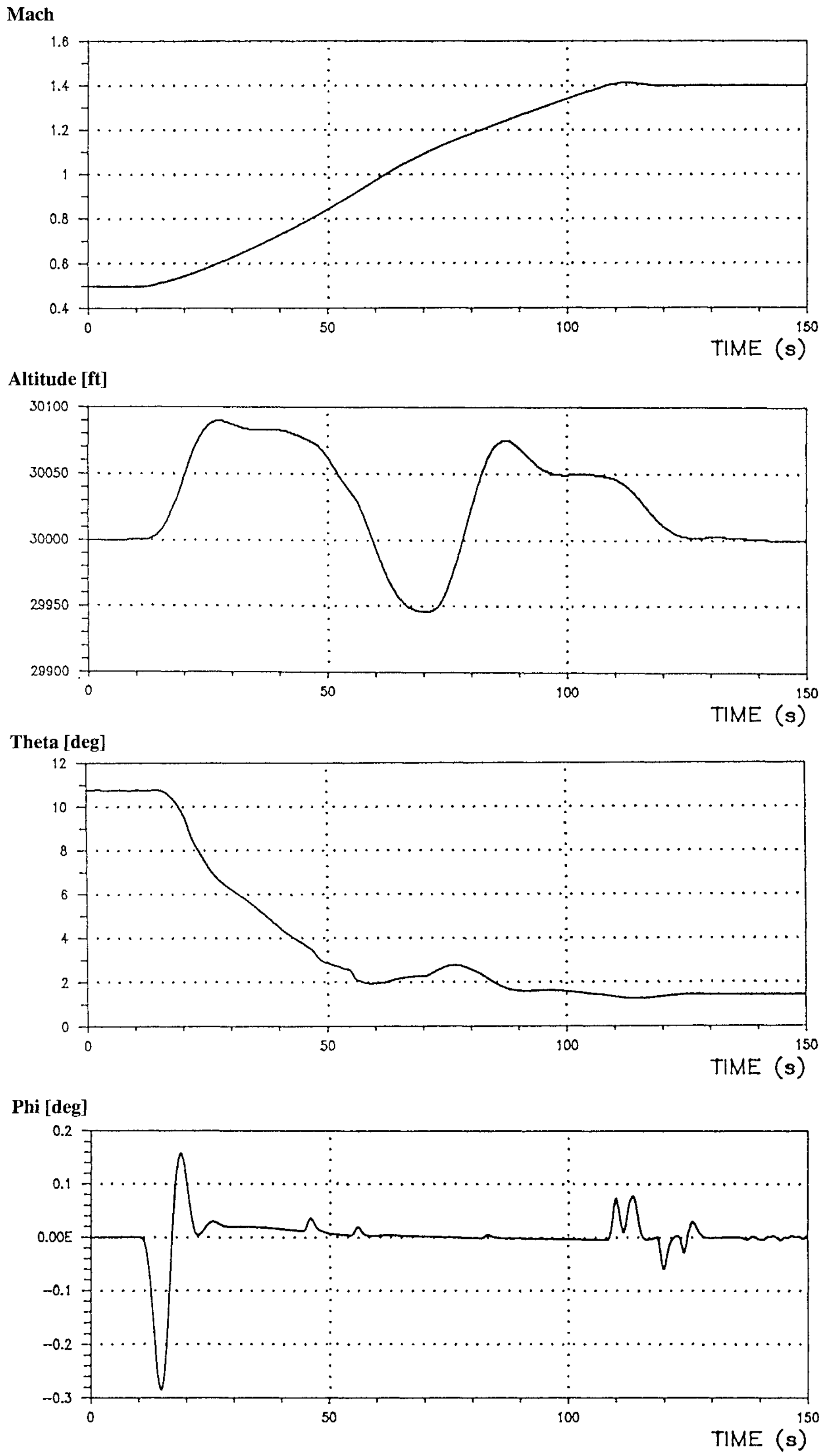


Fig. 9 Time histories: level acceleration.

the response to command input. The obtained PIs at four evaluation points show that the control law is conservative. In order to obtain the maximum agility as a fighter airplane, some sort of sophistication such as gain scheduling, smaller delay time margin, full-output feedback, and additional feedforward-loop is necessary. In this paper, however, a constant-gain control law is proposed to show its feasibility as an autopilot through a wide range of flight conditions. The simple control is not due to the limit of the MDM/MDP approach, but it is merely a result of simplifying the design problem.

Minor Corrections for Practical Control Laws

The constant-gain control law obtained with the MDM/MDP approach is simple and practical, but there still remains a small gap between it and a practically implementable control law. Modifications are as follows:

1) Inertial vertical speed was reconstructed from motion sensors and inertial altitude.

2) Directly measured variables, such as pitch attitude, stabilator position, and throttle position, were compensated for by roughly estimated trimmed steady-state values.

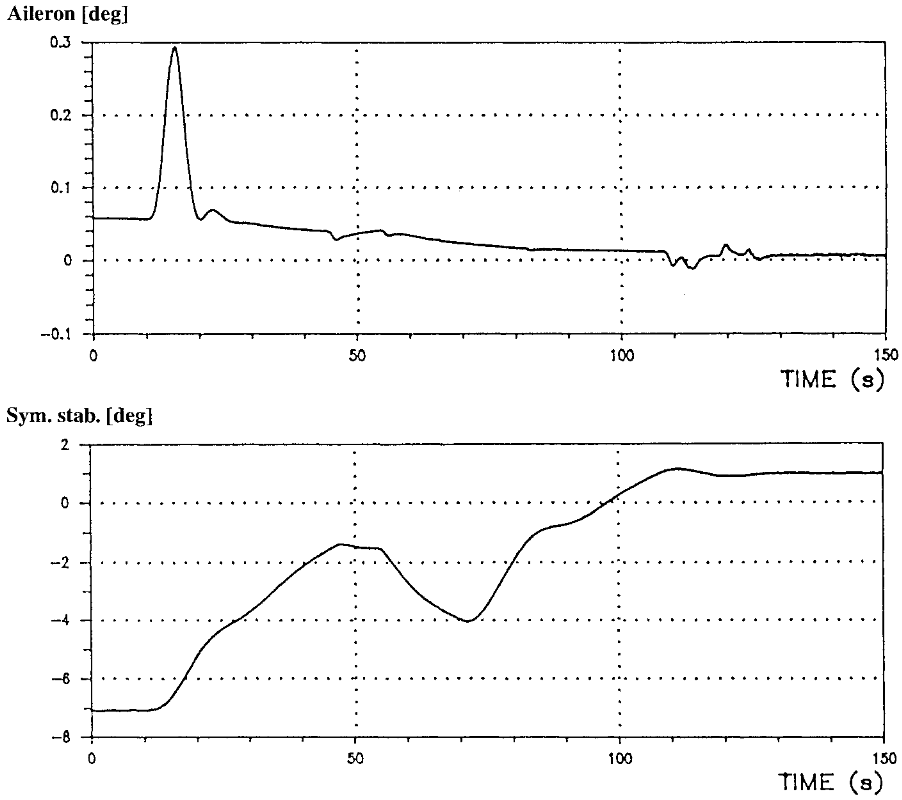


Fig. 9 (Continued) Time histories: level acceleration.

Table 3 MDM performance index and performance index of average delay time at four evaluation points

	Longitudinal		Lateral-directional	
Point	MDM PI	PI ($\frac{1}{2}$ delay)	MDM PI	PI($\frac{1}{2}$ delay)
A	11.33	11.08	2.53	2.49
B	13.99	13.53	5.39	5.09
C	11.49	11.19	2.27	2.19
D	unstable	10.62	1.78	1.65

3) Antiwindup limiters were implemented for time-integral variables, such as altitude, Mach number, roll attitude, and lateral acceleration error time integrals. Limiters for command errors were also added. Each limiter value is designed by trial and error and by rule of thumb.

4) The linear state equations are derived from level flight conditions. For large bank angle flight, outputs should be modified for longitudinal motion. Altitude error to be fed back was amplified by a factor of $1/\cos\Phi$, and pitch rate was replaced by $d\theta/dt$ that is reconstructed from the pitch rate, yaw rate, and roll attitude.

5) In the analysis, thrust is assumed as a control input, and engine dynamics is not explicitly modeled but is included in the MDMs. The gain of thrust to throttle position was simply assumed as 300 lb/deg in order to calculate the throttle position command.

These minor modifications were conducted for the final MDM/MDP control laws to be implemented in the simulation program. Simulations of two given flight tasks were performed with these final realistic control laws.

Level Acceleration at Mach 0.5–1.4

Time histories by numerical simulation with the final control law are shown in Fig. 9 for level acceleration flight. Responses of control variables and state variables of the aircraft are plotted. Concerning command inputs, $M_c = 1.4$ is commanded at $t = 10$ s, and maintaining altitude is another command, i.e., $H_c = 30,000$ ft. For lateral-directional motion, bank angle and lateral acceleration are explicitly controlled, i.e., $\phi_c = 0$, and $a_{yc} = 0$, but heading angle and lateral flight path are not explicitly controlled. Altitude error varies from −55 to 90 ft, which exceeds ±50-ft specification. Lateral-directional motion is not significantly affected in this flight. At Mach 1.4 there remains a small-amplitude limit cycle in the thrust. This problem may be due to insufficient modeling of the engine dynamics in the control design when using the afterburner.

3G Coordinated Turn

A 3 g turn flight simulation was performed with the final control law. Figure 10 shows its time histories. Responses of control and state variables are plotted. Concerning command inputs, the bank angle corresponding to normal 3 g, i.e., $\phi_c = 71.565$ deg, is commanded at $t = 10$ s, and $a_{yc} = 0$ is also commanded for the coordinated turn condition. Constant altitude and velocity are commanded for longitudinal motion, $H_c = 10,000$ ft, $M_c = 0.8$. The maximum altitude loss is 32 ft, and the maximum Mach loss is 0.008. A small-amplitude oscillation in pitch is observed in the steady turn, which is caused by a limited resolution of the altitude measurement.

Discussion

Merits of the MDM/MDP approach have been discussed in the Introduction. Therefore, the weak points and problems of the MDM/MDP are mainly discussed here.

1) The MDM/MDP is a linear structured feedback controller. As for the control structure, the LQR-type full state feedback is considered to be the most complex one, and only proportional feedback of outputs corresponding to full states is assumed. Some trials and errors would be necessary in order to find the appropriate gain structure, or partial-output feedback. Furthermore, the design is based on the linear system analysis, and this causes a limitation in introducing a realistic controller, such as design of limiter values for command errors and their time integrals.

2) Design goals should be represented with a PI. For flight control design, PIs that represent most of the design goals can be simply defined based on kinematics. However, this is not always guaranteed.

3) Although delay time is an important design parameter presenting uncertainty of the control input response, it could only be roughly estimated. The delay consists of actuator linear/nonlinear dynamics,

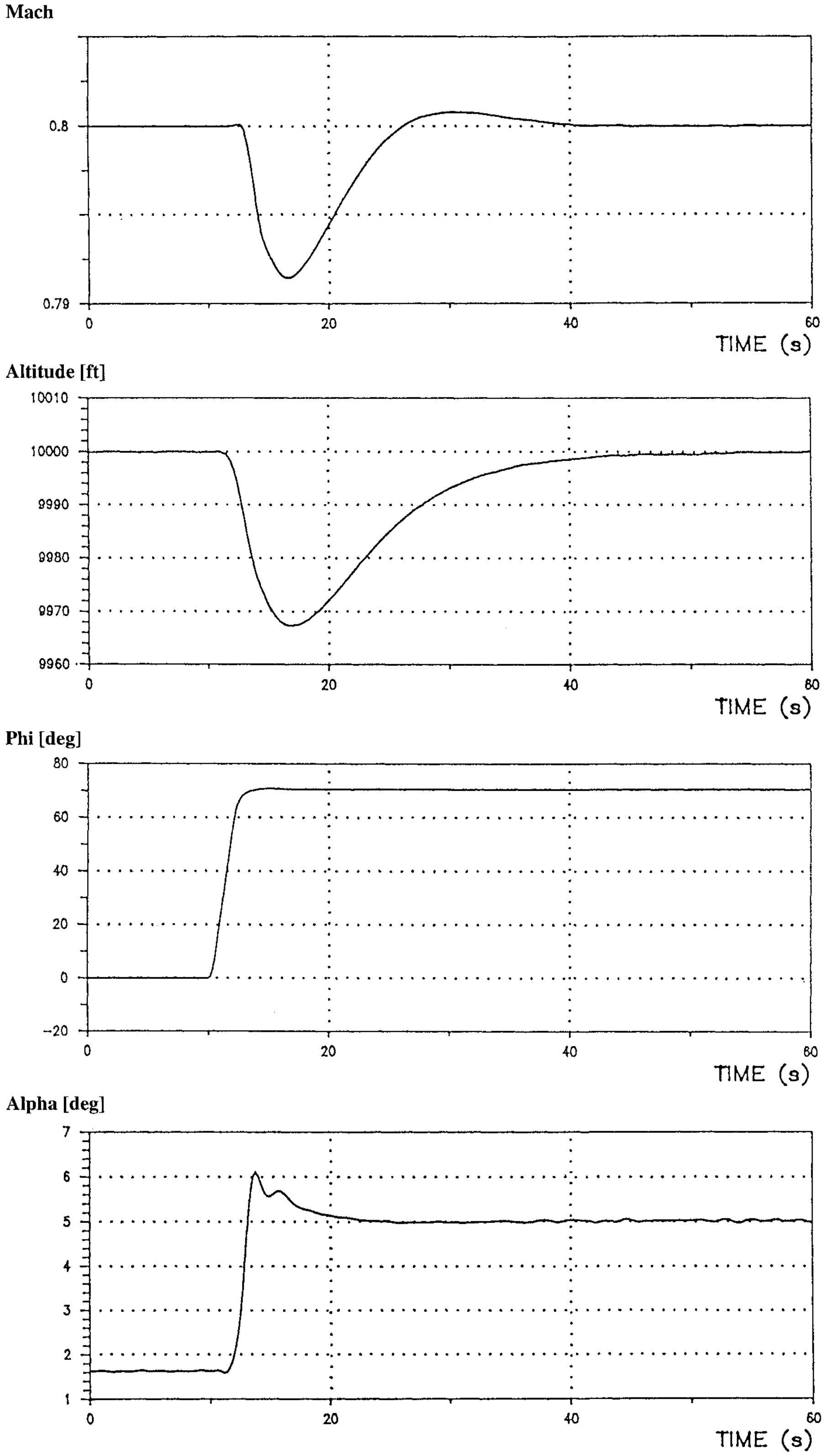


Fig. 10 Time histories: 3 *g* coordinated turn.

signal processing, sensor dynamics, filters, and miscellaneous uncertainties. For aerodynamic control surfaces, it is estimated to be around 0.2 s; so the design was carried out with 0.4 s as a result of some additional margin. For the thrust control, 2 s is used for the MDM approach. When using an afterburner, the engine dynamics are different, but both cases are treated with the same delay model.

4) With the MDM approach, it is not necessary to take into account control cost weighting matrices, but there are other design parameters, such as quadratic PI weighting matrices, initial condition covariance matrices, and weighting parameters on multiple models. It has been determined through numerical applications that simply defined weighting matrices and initial conditions are feasible, and they do not need to be adjusted. Some sort of trial and error, however, would be necessary for more sophisticated flight control.

5) Analysis was conducted with continuous systems. Since the control law structure is very simple, there is no problem in continuous discrete transformation: e.g., proportional output feedback is the same, and the time integral of command errors and filter dynamics are realized with straight transformations. Since the control law is simple, there is no problem in real-time computation for it.

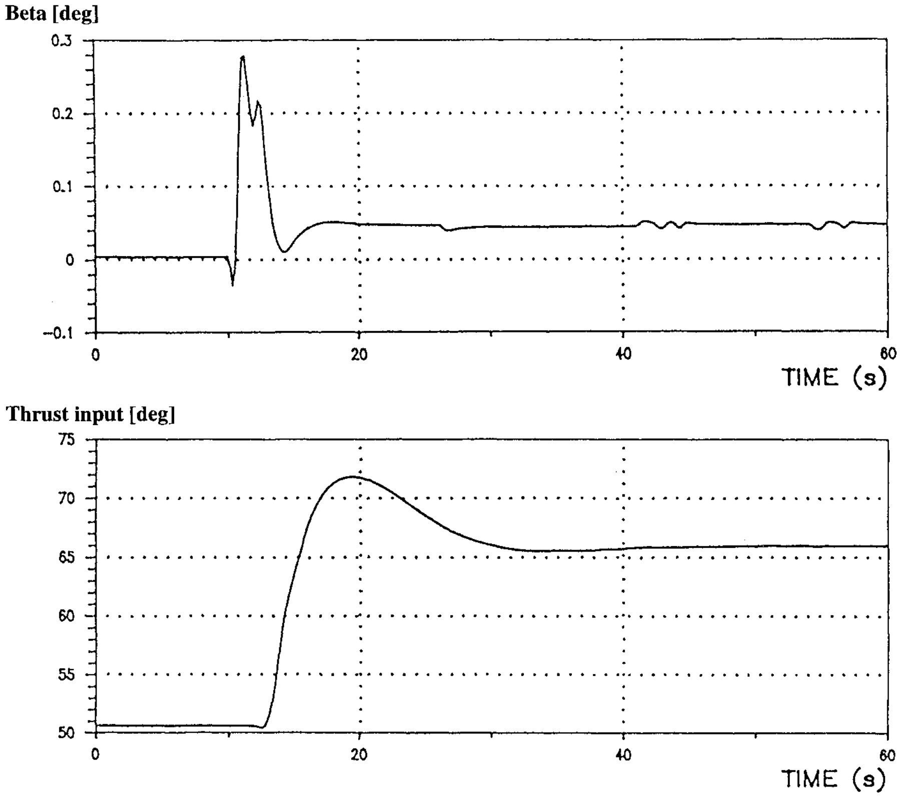


Fig. 10 (Continued) Time histories: 3 g coordinated turn.

Concerning the final control law, the author does not intend to claim its completeness or optimality. Since the assumed control structure is quite simple and the feedback gain is constant, the result obtained may be conservative. There may be room for additional refinement concerning such issues as scheduled gain control law, flexible gear ratio of differential stabilator to aileron, and engine dynamics modeling for thrust control input.

Concluding Remarks

The MDM/MDP approach is applied to the AIAA 1992 Controls Design Challenge. The MDM approach is used to ensure the robustness against unstructured uncertainty, or high-frequency uncertainty. The MDP approach using adequately selected different flight conditions is also used to ensure the performance robustness through a wide range of flight conditions. A constant-gain feedback control is finally derived with this approach. It is compared with the more complex control laws designed at each design point. The performance degradation due to constant gain is limited, and the constant-gain controller is feasible. The control law is checked with time histories of two flight simulations and the performance is evaluated at the four given points.

The MDM/MDP approach belongs to analytical control design methods, where the PI and measure of the system robustness should be appropriately defined. Control performance vs delay in the control input is an appropriate measure for control system robustness, and MDMs can be used in the optimal control. In practice, this approach gives reasonable control laws when the output feedback control structure is predetermined and control laws are introduced with parameter optimization. The PI is easily defined, and uncertainty and change of dynamics are easily represented with multiple models, and the design can be carried out with a small number of trials and errors.

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